Network Analysis

Introduction:

Network analysis has played an important role in electrical engineering. Application of network analysis have been made in information theory, the study of transportation problems and the planning and control of research and development projects. In transportation problem, there are number of routes to each terminal. But we like to choose a route of which the cost (or time) is minimum i.e., there is a problem to select the shortest route through a network. Thus the problem of network is to find a course of action, which minimizes some measure of performance.

Theory of Graphs:

A graph consists of a set of function point called “nodes” with certain pairs of the nodes being joined by lines called “branches” (or arcs, or links or edges). We shall denote the nodes of the graph by numbers 1, 2, 3, …….. Any branch of a graph can be characterized by the pair of nodes which it joins. Any two nodes can be connected by more than one branch. The two nodes joined by a branch are referred to as the end points of the branch. The degree of a node is the number of branches of the graph which have this node as an end point. A node of degree one is called an extreme point of the graph. A branch one of whose end points has degree one is called terminal branch. A branch of a graph is said to be oriented (or directed) if there is a sense of direction attributed to the branch so that one node is considered the point of origin and node j as the point of termination can be represented algebraically by $\overrightarrow{(i,\ j)}$and the non terminal branch joining nodes i, j is represented by (i, j)

Few Important Definitions:

Oriented Graph:

A graph in which all the branches are oriented is called an oriented graph.

Path:

A path joining nodes i and j is a sequence of branches connecting these two nodes. The nodes i, j are called the extremity points of the path.
Two different paths (chains) joining nodes 1 and 5 may be given as follows.
(a) (1, 2), (2, 3), (3, 6), (6, 4), (4, 5)
(b) (1, 2), (2, 6), (6, 5)

Loop or Cycle:

If the extremity points of the path are one and same node, then the path is called a loop
Thus, the chain (1, 2), (2, 4), (4, 1) is called a loop (or cycle).

Connected Graph:

A graph is said to be connected graph if there is a path (or chain) connecting every pairs of nodes of the graph.

Tree:

A tree is a connected graph which has no loops. In other words a connected graph is a tree if and any if the path joining any two nodes is unique. A tree always has at – least two extreme points.

Network:

A network is a graph such that a flow can take place in the branches of the graph.
If an oriented graph is a network, the orientation to branch is assumed to be the feasible direction of the flow along the branch. A network need not to be oriented since it may be feasible to have flow in either direction along a branch. If there is a limit to the magnitude of the flow in any branch of a network, then a capacity restriction is imposed on that branch. The flow capacity of a branch in a specified direction in the upper limit to the feasible magnitude of the rate of the flow (or total quantity of flow) in the branch in that direction. A branch is oriented if the flow capacity is zero one direction.
A node j in a network is called a source if every branch which has this node as end point is oriented in such a way that the flow in the branch moves away from this node j. Similarly a node i in a network is called sink if every branch which has this node as an end point is oriented in such a way that the flow is from other nodes to this node i.
In a network, we assume that branches are connected only at nodes. For example if the branches (1, 2), (3, 4) cross, then it is assumed that they are not interconnected i.e., the flow in branch (1, 2) can not enter the branch (3, 4) at the point of intersection.

The problem of network analysis can be dealt by following methods

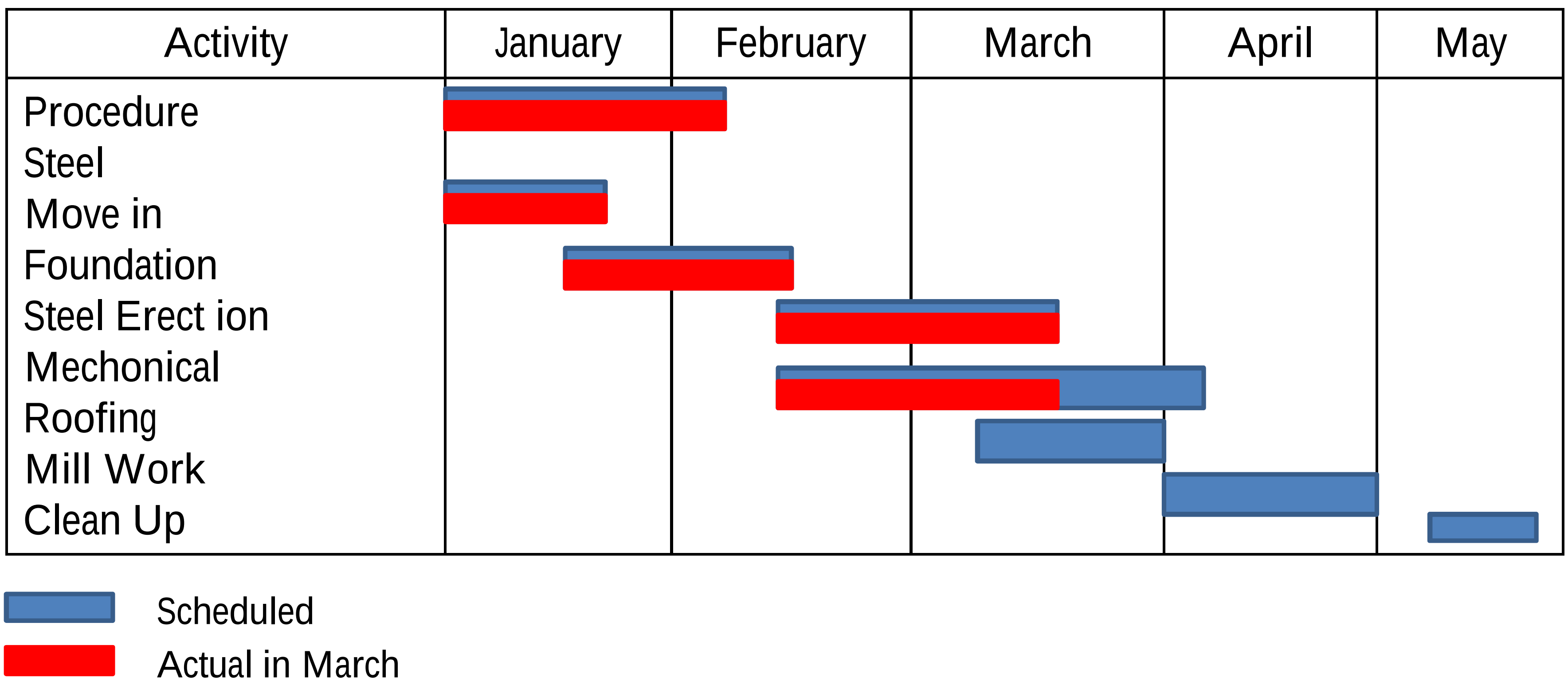
(i) Schedule Chart

(ii) Critical Path Method (CPM)

(iii) Project Evaluation and Review Technique (PERT)

Schedule Chart (Gantt Bae Chart):

Until three decades ago, the Gantt bar chart was used for studying the planning of projects. It specifies the start and finish times for each activity on a horizontal time scale.



It is clear from the chart that the work is in progress up to the month of March and four operations, Procure steel, Movein, Foundation and steel creation are not behind the schedule. Only one operation mechanical is behind the schedule. Thus seeing the chart the contractor will be confident (in march) of the completion of the work in the time.

Disadvantages of the Gantt bar Chart:

The main disadvantage of the Gannt's Chart is that the independence between the different activities controlling the progress of the project can not be determined from this chart.

Construction of the Network Diagram:

The following questions will generate the necessary information for the construction of a network diagram:

1. Which operations (or activities) must be completed immediately before each operation (or activity) can start.
2. Which operations can be carried out simultaneously.
3. Which operation follow other given operations.

For the construction of a network diagram the following rules are followed:

Rule 1. Each operation (or activity) is represented by one and only one arrow in the network. (This rule ensures that none of the operations can be represented more than once in the network.)

Rule 2. No two operations can be identified by the same end events (nodes).

If two operations have same end events then we introduce a dummy operation (or dummy activity) between any one of the operations and one of the end events. It should be noted that a dummy operation does not consume any time or resources.

General Procedure of Construction of a Network Diagram:

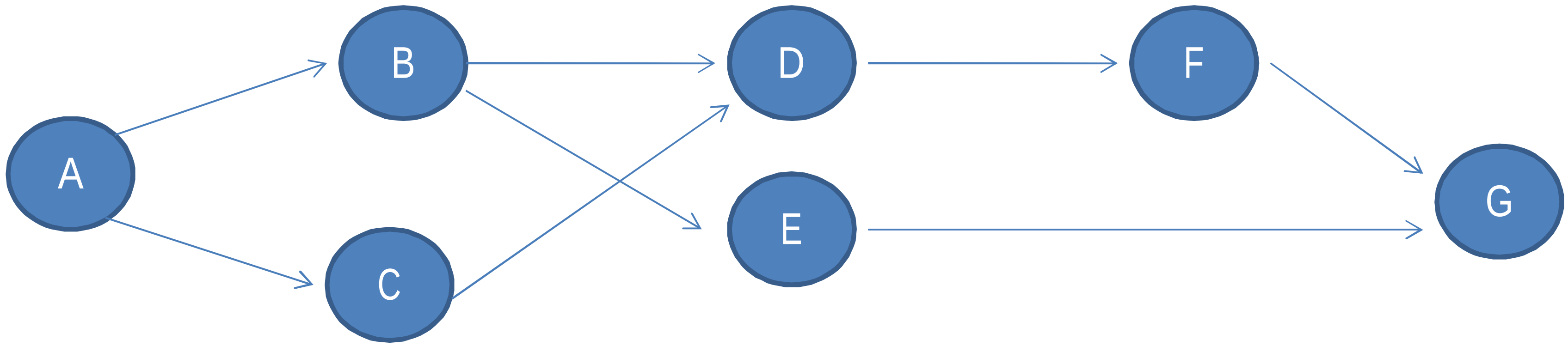
1. First of all prepare a list of operations and the preceding operations or the post operations.
2. In the first column (vertical line) list (by using circles) all the operations that can be started immediately.
3. In the second column list the operations that can be started immediately after any of the first column operations is completed.
4. Draw the directed lines (arrows) between the circles (nodes) to indicate which operation follow other operation.
5. Lable each circle drawn to represent the description of the operation.
6. In the third and successive columns list the operations that can be started immediately after any of the proceeding column operation completed.
7. Proceed till all the operations are completed.
8. After completing the preliminary draft including all operations check for logical arrangements.
9. Try to avoid arrows which cross each other.
10. Use arrows from left to right but avoid mining two directions. Vertical arrows may be used if they are necessary.

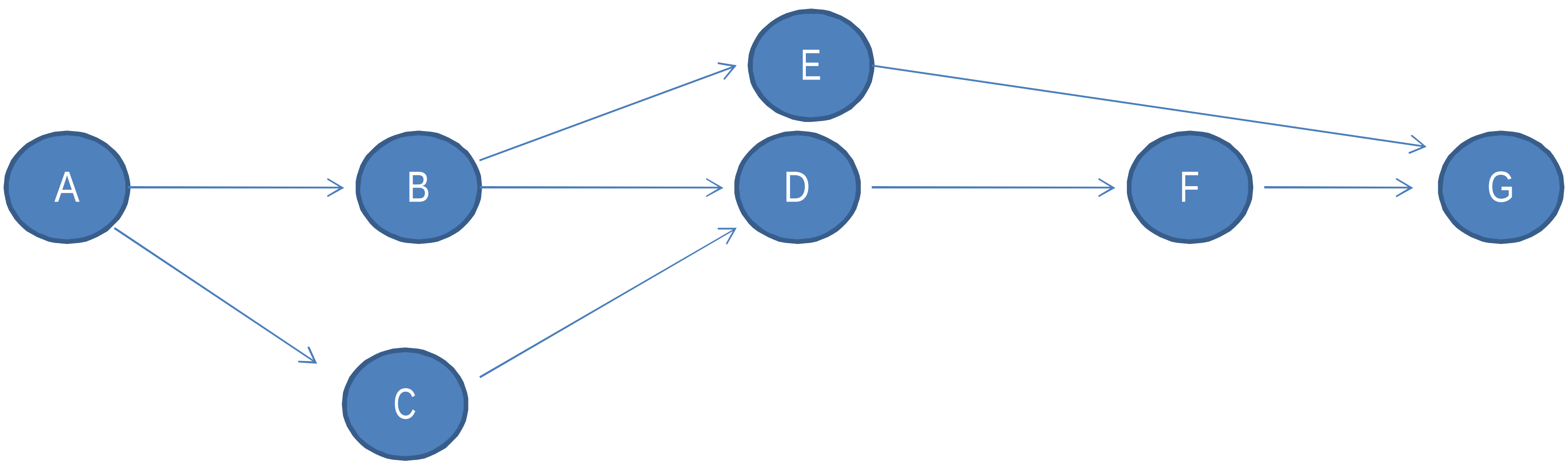
11. After getting the final draft place consecutive numbers in each circle beginning at the upper most circle of the first column and the number vertically downwards. After numbering first column begin from the upper most circle of the next column. Preceding in this way number all circles. It may be checked that no circle will follow a circle with large number.

Example 1.

Draw a network diagram for the following set of operations represented by separated latters.

Operations	Post – operations
A	Precedes B, C
B	Precedes D, E
C	Precedes D
D	Precedes F
E	Precedes G
F	Precedes G

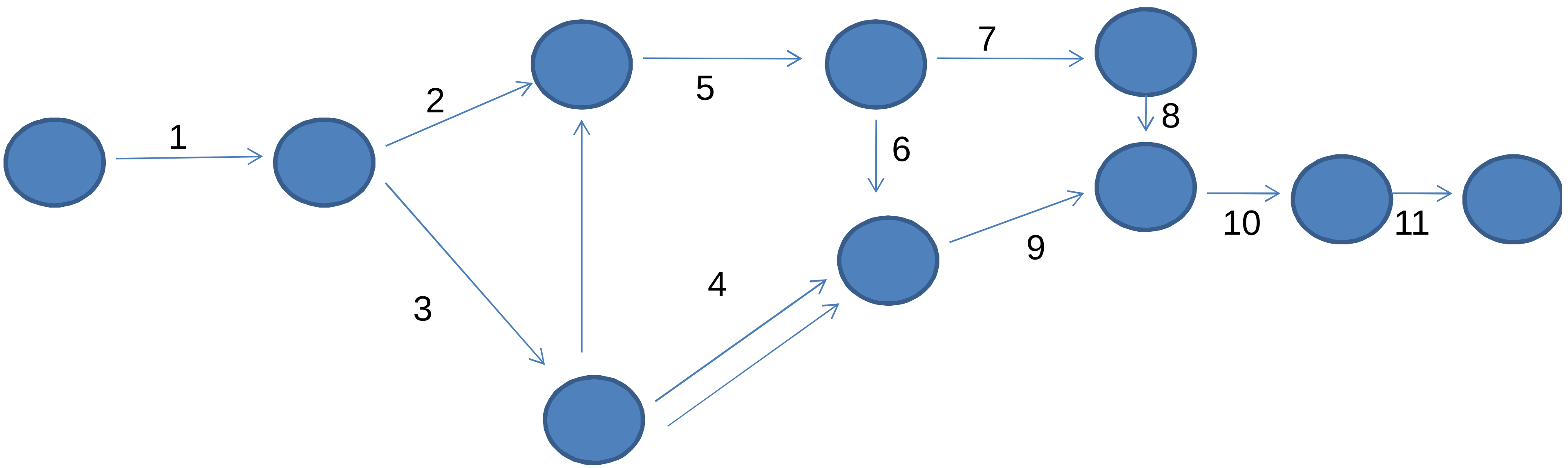

A
B
C
D
E
F
G


A
B
C
D
E
F
G

Example 1.

A new type of water pump is to be designed for an automobile. Prepare a network diagram from the following table showing the major activities for effective control of the project and the preceding activities.

No.	Activity	Post – operations
1	Drawing prepared and approved	-
2	Cost analysis	1
3	Tool feasibility (economics)	1
4	Tools Manufactured	3
5	Favorable Cost	2, 3
6	Raw Materials Produced	4, 5
7	Sub – assemblies ordered	5
8	Sub – assemblies received	7
9	Parts Manufactured	4, 6
10	Final Assembly	9, 3
11	Testing and Shipment	10


1
2
3
4
5
6
7
8
9
10
11

CPM / PERT Techniques:

The two techniques were develop simultaneously by two different groups in 1956 – 58. Walker from E.I. du Ponde Nemours company developed the critical path method (CPM) to solve project scheduling problem. Also the project Evaluation and Review Technique (PERT) was developed by a team of engineers working on Polar's Missile Programmes of the U S Navy. Both methods are network oriented techniques using the same principles. Both methods have found applications in Government organizations also. Such as:

(i) Construction of a building highway
(ii) Construction of a dam or canal system
(iii) Development of supersonic planes
(iv) Designing of a prototype of a machine, etc.

Critical Path Method:

In critical path method firstly the network diagram is drawn and the required time to do each operation is written to the left of each operation circle. Then these times are combined to develop a schedule which minimizes or maximizes the measure of performance for each operation. The schedule thus obtained is used to determined the required critical path.

The critical path method can be used effectively in production planning, road systems and traffic schedules, communication network etc.

Determination of Critical Path Method:

After the network representation of the project, the next objective is to determine the critical path. One method of determination of such a path (critical path) is labeling method due to Minty. This method consist of building of a shortest route tree, one link at a time, working outwards from the origin. The property of a tree is that path between any two nodes is unique. The method is applied systematically as follows.

Step 1. label origin (starting node) with distance zero.

Step 2. Write all the links whose tail ends are labeled and whose heads are unlabelled and for each of these links find the sum of label at the tail end and its length. Select the link for which this sum is minimum (or maximum) and label the head of this link with this sum.

Step 3. Repeat step 2 of the next eligible link. The process terminals if either the terminal node is labelled or all nodes are labelled.

The links thus obtained will from a required shortest tree.

The Project Evaluation and Review Technique (PERT):

So far we have discuss the network for the projects in which the time values are deterministic. This is true for the projects such as maintenance of a machine, construction of building or road etc. However, there are projects such as research project, design of a new machine etc, in which the time values are not deterministic. Such projects are handled by PERT. The main objective in the analysis through PERT is to find out whether a job could be finished by a given date. In PERT we assume that the expected time of any operation can never be determined exactly. In this approach we use the following three time values associated with each operation (or activity).

(i) Optimistic Time: It is the shortest possible time in which the operation (or activity) can be completed if everything goes very well.

(ii) Most likely time: It is the estimate of the normal time the operation (or activity) would take.

(iii) Pessimistic Time: It is the longest time that an operation (or activity) could take under adverse condition.

Let, t_0 = Optimistic Time

t_m= Most likely Time

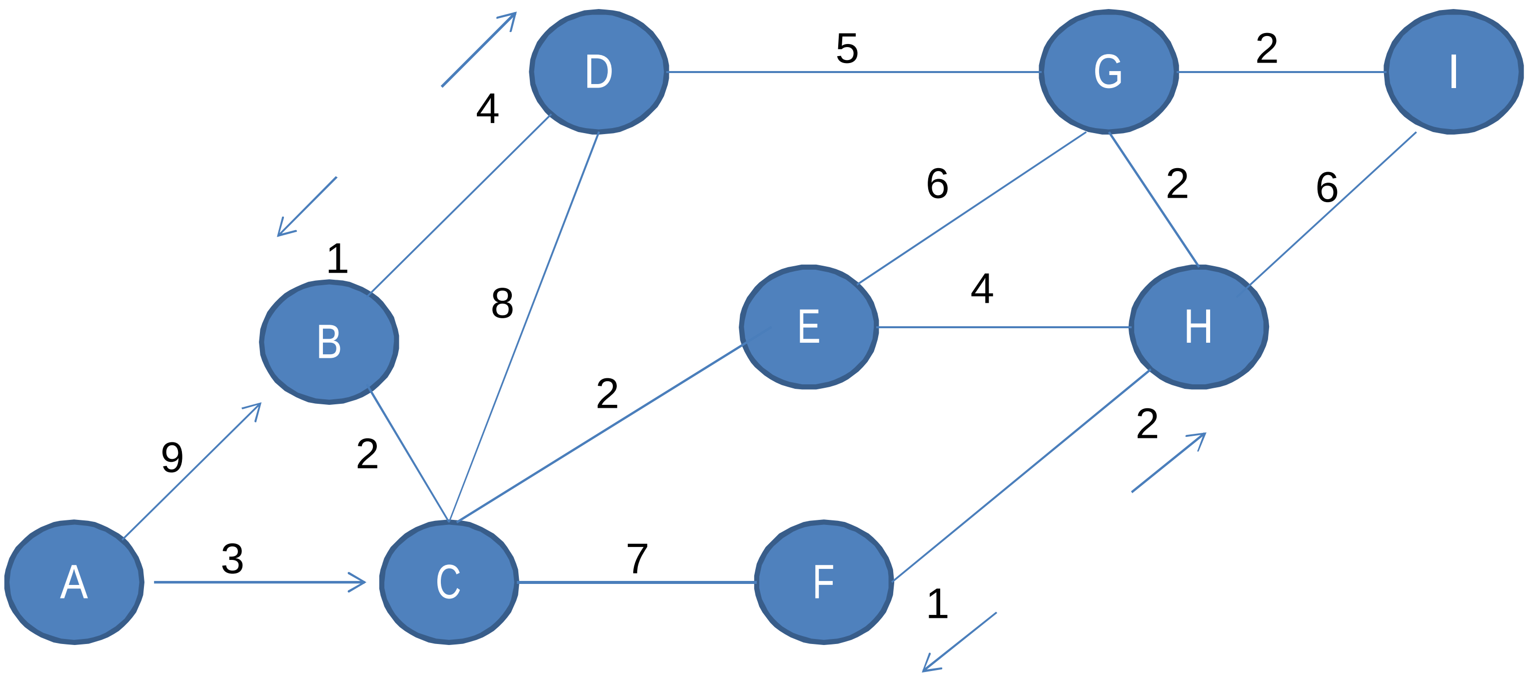
and t_p = Pessimistic Time

Then the expected value (t) connected with any operation (activity) is given by

$$t = \frac{t_0 + 4t_m + t_p}{6} \quad ....(1)$$

And the variance of t is given $V(t) = \left(\frac{t_p - t_0}{6} \right)^2$

Example: Find the critical path for following network:



Solution:

Step 1. Here A is the origin (starting node). Therefore we label A with distance 0.

Step 2. Now we write the links with labeled tail end A and unlabelled heads. For each of these links the sum of label at the tail lengths are as follows:

Links	Sum of Label	Links selected	Node labeled with value of label
AB	0 + 9 = 9		
AC	0 + 3 = 3*	AC	C(3)

Step 3. After labeling the node (end) C with 3, we write the links with labelled tail ends A, C and unlabeled heads. The links selected in this way and their sum etc, are as follows

Links	Sum of Label	Links selected	Node labeled with value of label
AB	0 + 9 = 9		
CB	3 + 2 = 5*	CB	B (5)
CD	3 + 8 = 11		
CE	3 + 2 = 5		
CF	3 + 7 = 10		

Step 4.

Links	Sum of Label	Links selected	Node labeled with value of label
BD	5 + 4 = 9		
CD	3 + 8 = 11		
CE	3 + 2 = 5*	CE	E(5)
CF	3 + 7 = 10		

Step 5.

Links	Sum of Label	Links selected	Node labeled with value of label
BD	5 + 4 = 9*	BD	D(9)
CD	3 + 8 = 11		
CF	3 + 7 = 10		
EG	5 + 6 = 11		
EH	5 + 4 = 9		

Step 6.

Links	Sum of Label	Links selected	Node labeled with value of label
DG	9 + 5 = 14		
CF	3 + 7 = 10		
EG	5 + 6 = 11		
EH	5 + 4 = 9*	EH	H(9)

Step 7.

Links	Sum of Label	Links selected	Node labeled with value of label
DG	9 + 5 = 14		
EG	5 + 6 = 11		
CF	3 + 7 = 10	CF	F(10)
HF	9 + 1 = 10		
DG	9 + 6 = 15		

Step 8.

Links	Sum of Label	Links selected	Node labeled with value of label
DG	9 + 5 = 14		
EG	5 + 6 = 11*	EG	G(11)
HG	9 + 2 = 11		
HI	9 + 6 = 15		

Step 9.

Links	Sum of Label	Links selected	Node labeled with value of label
GI	11 + 2 = 13*	GI	I(3)
HI	9 + 6 = 15		

Hence the optimal path is

A $\longrightarrow$ C $\longrightarrow$ B $\longrightarrow$ E $\longrightarrow$ D $\longrightarrow$ H $\longrightarrow$ F $\longrightarrow$ G $\longrightarrow$ I

With distance equal to 13